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# Fundamentals of Sintering

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Textbook: “Sintering: densification, grain growth and microstructure” Elsevier (2005)

Supporting materials: selected papers and book chapters

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## **Main Sequence:**

### **Part I. Basis of Sintering Science**

- Brief description of sintering processes and their parameters
- Interfacial energy and driving force of sintering
- Sintering and polycrystalline microstructure

### **Part II. Bonding and Densification**

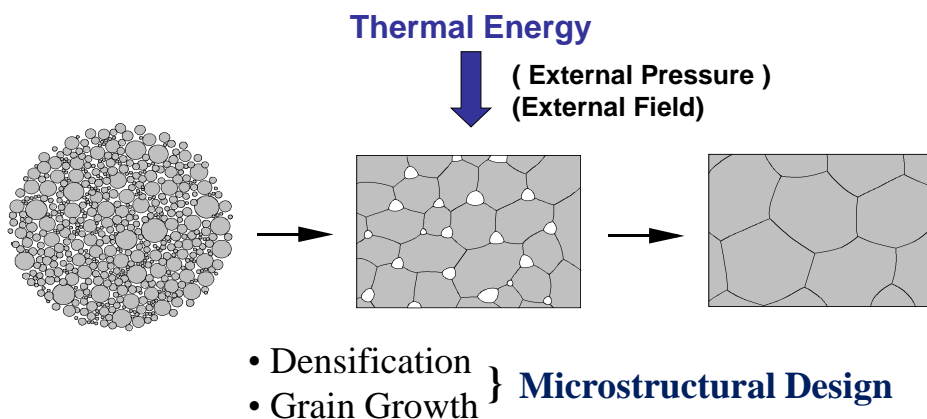
- Solid state sintering (SSS) Models and Densification
  - Models and kinetics
  - Effects of processing variables
- Liquid phase sintering (LPS) Models and Densification
  - Role of liquid in densification
  - Densification kinetics (effects of processing variables)

### **Part III. Grain Growth and Microstructural Evolution**

- Liquid phase sintering
  - Grain growth in a matrix (Ostwald ripening)
  - Effect of interfacial energy anisotropy
- Solid state sintering
  - Grain growth in a pure dense system
  - Effects of second phase particles and solute segregation
  - Effect of pores on microstructure development
  - Effect of boundary energy anisotropy

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# What is Sintering?



A processing technique of materials to produce density and microstructure controlled materials and components from metallic or ceramic powders by applying, in general, thermal energy.

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## Sintering:

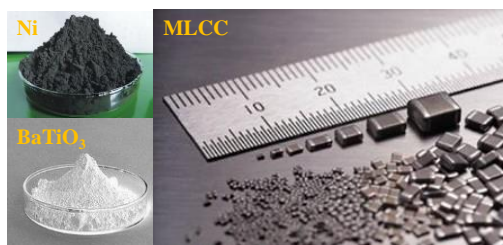
the **oldest**, but probably an **everlasting technique**  
 in materials and components fabrication

**Prehistoric era**



**Firing of pottery**

**21<sup>st</sup> century**



**Fabrication of MLCC**

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# PI: Basis of Sintering Science

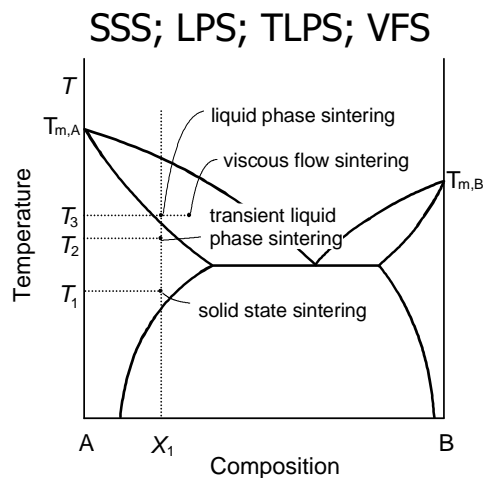
## Outline:

- Brief description of sintering processes and their parameters
- Interfacial energy and driving force of sintering
- Characteristics of polycrystalline microstructure

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## Chap. Sintering Processes

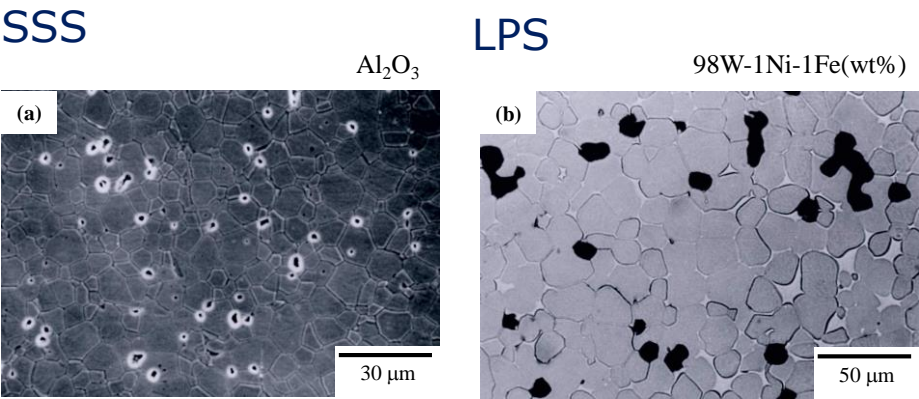
### Categories of sintering:



S.-J. L. Kang, "Sintering : Densification, Grain Growth and Microstructure", Elsevier, Oxford (2005).

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# Typical microstructures observed during SSS and LPS



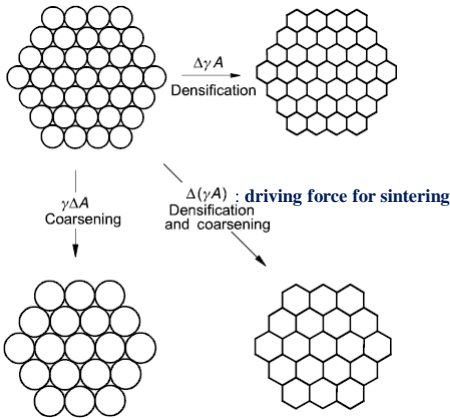
S.-J. L. Kang, "Sintering : Densification, Grain Growth and Microstructure", Elsevier, Oxford (2005).

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## Driving Force for Sintering

### Reduction of the total surface energy

0.5 ~ 500J/mole (100μm ~ submicron size)  
(cf. chemical free energy of compound formation)



Note: the meaning of sintering

S.-J. L. Kang, "Sintering : Densification, Grain Growth and Microstructure", Elsevier, Oxford (2005).

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## Exercise:

- Energy change with the sintering of cube-shaped powder with an edge of  $l$  with no grain growth

$$6L^2\gamma_s \frac{1}{L^3} \rightarrow \frac{6}{2}L^2\gamma_b \frac{1}{L^3}$$

$$\Delta E = \frac{6}{L} \left( \frac{\gamma_b}{2} - \gamma_s \right)$$

- Meaning of  $\gamma_s$  and  $\gamma_b$
- Sintering of SiC (addition of C and B)

## Qn:

Total surface energy = f(particle size)

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## Densification and Coarsening

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- The two processes are themselves complex.
- Multiple mechanisms can be operative for each process (parallel processes) cf. serial processes
- Simultaneous and mutually interactive processes (The choice of processing condition is important)
- We usually want to prepare materials with high density and fine grain size by increasing the densification rate relative to coarsening rate.

$$(\dot{\rho}/\dot{G}) \uparrow$$

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## Directions for attaining $(\dot{\rho}/\dot{G})\uparrow$

- Modification of powder characteristic  
(shape, size and size distribution, etc.)
- Modification of chemistry
  - Use of additives in solid solution    **Why?**  
eg)  $\text{Al}_2\text{O}_3 + \text{MgO}$ ,  $\text{WC-Co} + \text{VC}$
  - Use of a second phase, commonly liquid
  - Atmosphere control ( $\text{Po}_2$  control)    **Why?**  
eg)  $\text{BaTiO}_3$ ,  $\text{SrTiO}_3$ , Ni, Cu
- Modification of the sintering process  
Unconventional sintering processes  
eg) HP, HIP, Fast Firing, Two-step sintering, SPS,  
Flash sintering, Cold sintering

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## Sintering Variables

- i) Variables inherent to raw powders (material variables)
  - **Powder** : shape, size, size dist, agglomeration (hard and soft), mixedness
  - **Chemistry** : composition, impurity, nonstoichiometry-atmosphere, homogeneity
- i) Variables related to sintering condition (process variables)
  - **T, t, P, atmosphere, heating and cooling rate, E, B**  
(magnetic field)
    - eg) P: HP, GPS, HIP and SPS(with E)
    - eg) E: SPS and Flash sintering
    - eg) Atmosphere :  $\text{Po}_2$  (oxidizing or reducing), inert

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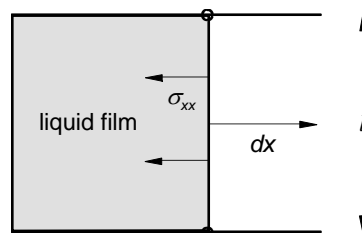
## Chap. Thermodynamics of the Interface

- Qn: Why does bonding occurs between particles?
  - (i) Two particle model, (ii) Surface and Geometry
  - (iii) Driving force (difference in chemical potential)

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## Surface Tension (stress) and Surface Energy

Qn: How do we know that there is a tension force on a surface?



$$\text{Work : } W = 2\sigma_{xx}l dx$$

$$\text{Energy : } \Delta E = \Delta A \cdot \gamma = 2\gamma l dx$$

$$\sigma_{xx} = \gamma$$

$$\sigma = (\sigma_{xx} + \sigma_{yy})/2 = \gamma$$

$\sigma$  // to the surface

$\gamma \perp$  to surface

Nature of surface tension:

surface configuration of atoms

Qn: Cases of liquid and solid

Qn: Case of a thin film or 2-D material

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## *Relation btw. $\gamma$ and $\sigma$*

- $\sigma_{ij} = \delta_{ij}\gamma + d\gamma/d\varepsilon_{ij}$

$\delta_{ij}$  : Kronecker's delta

for  $i = j$ ,  $\delta_{ij} = 1$

for  $i \neq j$ ,  $\delta_{ij} = 0$

- The equality of  $\sigma$  and  $\gamma \leftarrow$  mobility of atoms

- Driving force of sintering

Thermodynamics:  $\Delta(\gamma A)$

Kinetics:  $\sigma$  and geometry

- Case of sintering

$T_{\text{sinter}}$  is usually  $\sim 2/3 T_m$ .  $T_{\text{sinter}} \downarrow$  as  $a \downarrow$ .

At usual sintering temperatures,  $\gamma \approx \sigma$ .

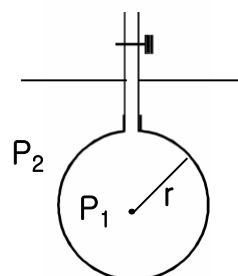
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## Thermodynamics of curved interface

Qn: Pressure difference btw two adjacent phases with a curved interface

- Blow of a soap bubble
- Inflate a balloon
- Water droplet and Gas bubble

### Curved interface



$$PdV = \gamma dA$$

$$P_1 - P_2 = \frac{2\gamma}{r}$$

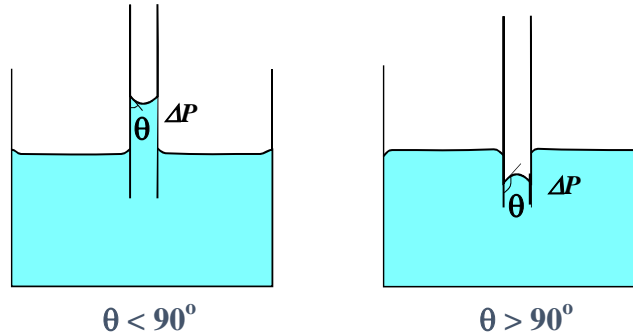
**Young-LaPlace Eq.**

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## Capillary Pressure

### Capillary action of a glass tube



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## Capillarity and Chemical Potential

System with two incompressible phases that are separated by a curved interface

$$\begin{aligned} d\Omega &= 0 = d\Omega^\alpha + d\Omega^\beta + d\Omega^\sigma \\ &= -P^\alpha dV^\alpha - P^\beta dV^\beta + \gamma dA \end{aligned}$$

$$\begin{aligned} P^\alpha - P^\beta &= \gamma \frac{dA}{dV^\alpha} \\ &= \gamma K \quad K = \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \end{aligned}$$

$$P^\alpha - P^\beta = \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \gamma$$

Cf: Energy change of compressible fluid due to a curved interface

**Deformation energy**

$$\begin{aligned} W &= - \int_0^P P dV = - \int_0^P P \left( \frac{\partial V_m}{\partial P} \right)_T dP = V_m \kappa \int_0^P P dP \\ &= \frac{1}{2} V_m \kappa P^2 \end{aligned}$$

$$\begin{aligned} \mu_r^\alpha &= \mu_\infty^\alpha + \gamma K V_m^\alpha \\ \mu_r^\beta &= \mu_\infty^\beta \end{aligned} \quad \text{Gibbs-Thompson Eq. Only to phase } \alpha$$

Freezing point of liq. ↓    Melting point of fine powder ↓

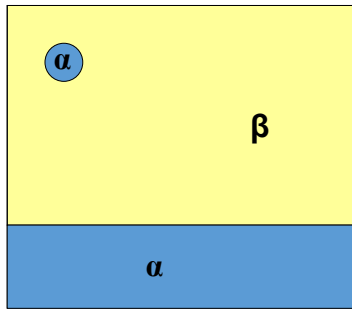
For  $r < 10^{-8} \text{m}$ , the size effect becomes significant ( $\sim 0.1$ ).

For  $r > 0.1 \mu\text{m}$ , the size effect is insignificant.

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## Capillarity and Atom Activity

### Curved Interface and Solubility



$$a_r = a_\infty \exp\left(\frac{2\gamma V_m}{RT r}\right)$$

$$a_r = a_\infty \left(1 + \frac{2\gamma V_m}{RT r}\right)$$

**Gibbs -Thompson Eq.**

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## Condensed and Dispersed Phase

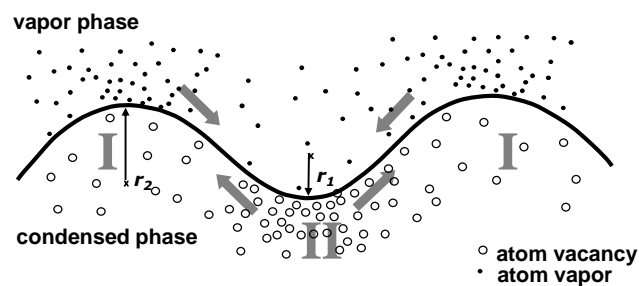
$\alpha$  : a condensed phase

$\beta$  : a dispersed phase

$$P^\alpha - P_\infty = (2\gamma/r) \cdot V_m^\beta / (V_m^\beta - V_m^\alpha) = 2\gamma/r \quad \text{for condensed phase}$$

$$p^\beta - p_\infty = (2\gamma/r) \cdot V_m^\alpha / (V_m^\beta - V_m^\alpha) = p_\infty (1 + 2\gamma V_m^\alpha / RT r) \quad \text{for dispersed phase}$$

*As P increases, p increases. (equality of  $\mu$ )*



Differences in i) Pressure, ii) Vacancy concentration, and  
iii) Vapor pressure (solubility).

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# Chap. Polycrystalline Microstructure

A few Questions:

Qn: What are the factors that affect microstructure and microstructural evolution?

Qn: What are the characteristics of a polycrystalline microstructure?

Representative (equilibrium) shape of grains?

A stationary microstructure?

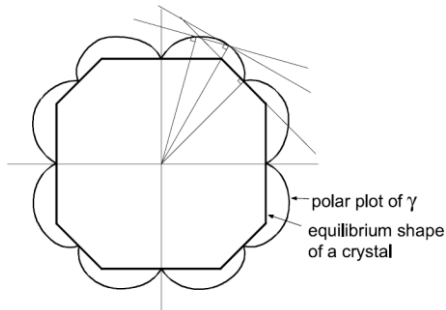
- Polycrystal: an Aggregate of Single Crystals

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## Equilibrium Shape of a Single Crystal

Equilibrium Shape:  $\sum (\gamma_i A_i)_{\min}$

Polar plot of surface energy  $\gamma$  ( $\gamma$ -plot)



**Wulff construction**

( $\gamma$  of a plane  $\propto$  the distance from the center to the corresponding  $\gamma$  point)

Herring, Phys. Review, 82, 87 (1951)

Kang, in *Sintering: Densification, Grain Growth and Microstructure*, Elsevier, Oxford (2005)

### Wulff Theorem

Under equilibrium

$$dF = \sum_i \gamma_i dA_i + \left( \frac{\partial F}{\partial n^c} \right)_{T,V} dn^c + \left( \frac{\partial F}{\partial V^c} \right)_{T,V} dV^c + \left( \frac{\partial F}{\partial n^s} \right)_{T,V} dn^s + \left( \frac{\partial F}{\partial V^s} \right)_{T,V} dV^s = 0$$

$$\sum \gamma_i dA_i + (\mu^c - \mu^s) dn^c - (P^c - P^s) dV^c = 0$$

$$\sum_i \left[ \gamma_i - \frac{h_i}{2} (P^c - P^s) \right] dA_i + (\mu^c - \mu^s) dn^c = 0$$

$$P^c - P^s = \frac{2\gamma_i}{h_i} \equiv K_w$$

$K_w$ : Wulff constant

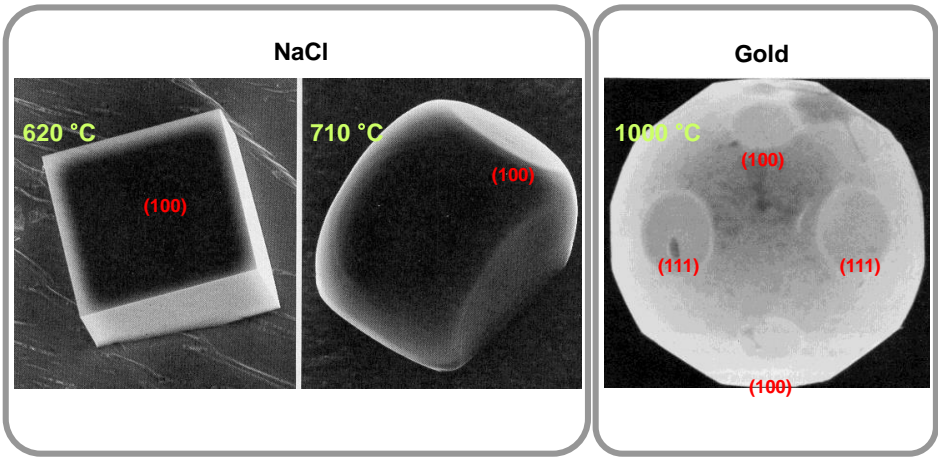
$$\mu = \mu^o + \frac{2\gamma_i V_m}{h_i}$$

cf. Gibbs-Thompson eq.

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# Equilibrium Shape of Single Crystals

## Examples



Simple calculation of surface energy using the broken bond model

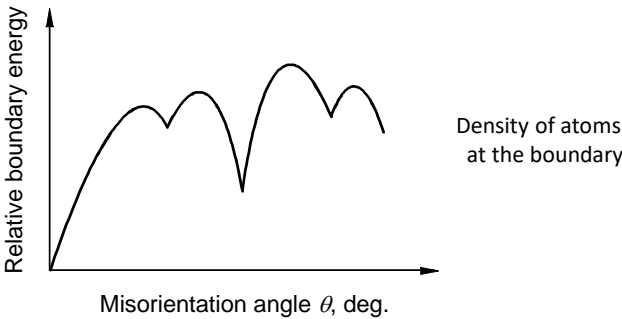
$$\gamma_{\theta} = \frac{U_b}{2a}(\cos \theta + \sin \theta) = \frac{U_b}{a\sqrt{2}}\cos(\theta - \frac{\pi}{4})$$

Heyraud and Métois, *J. Crystal Growth*, **84**, 503 (1987).  
Heyraud and Métois, *J. Crystal Growth*, **50**, 571 (1980).

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# Single Phase Microstructure

$$\gamma_b = f(\text{orientation})$$



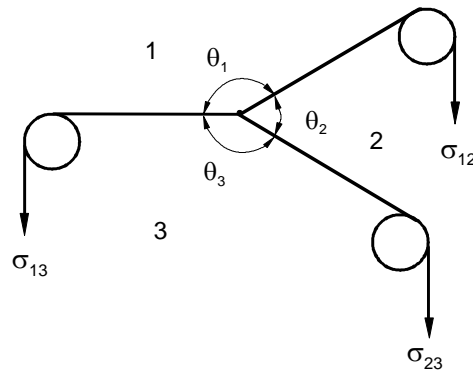
Simplification :  $\gamma_b = \text{const.}$  (a foam structure)

- Grain shape is determined by
- i) Local geometrical requirement  
(minimization of the total interface energy and the equilibrium condition of surface tensions)
  - ii) Overall requirement of space filling

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# Interfacial Tension and Microstructure

Equilibrium state btw three interfacial tensions

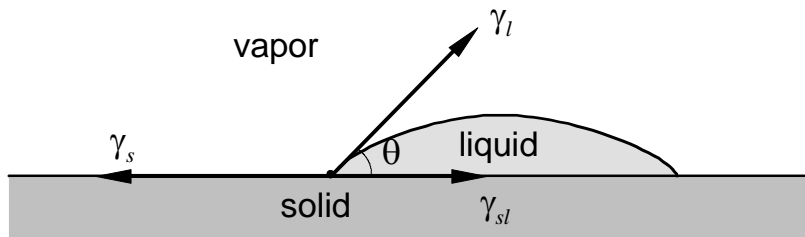


The sine law is satisfied. 
$$\frac{\sigma_{12}}{\sin \theta_3} = \frac{\sigma_{23}}{\sin \theta_1} = \frac{\sigma_{31}}{\sin \theta_2}$$

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## Wetting Angle

3-phase system, s/l/v



$$\gamma_s = \gamma_{sl} + \gamma_l \cos \theta$$

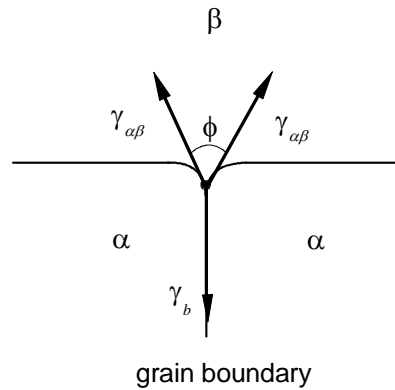
Complete wetting at  $\theta = 0$

$$\gamma_s \geq \gamma_{sl} + \gamma_l$$

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## *Dihedral Angle*

2-phase system, s/v or s/l



$$\gamma_b = 2\gamma_{\alpha\beta} \cos \frac{\phi}{2}$$

grain boundary

Usually,  $\gamma_s > \gamma_b$   $\phi > 120^\circ$

$\gamma_s \approx 3\gamma_b$   $\phi \sim 160^\circ$

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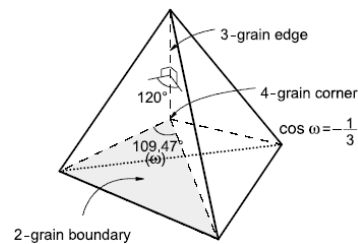
## *Equilibrium Grain Shape*

2-dim: hexagon      consider triangle and square

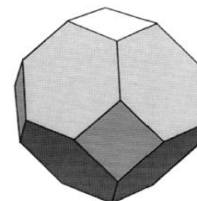
3-dim:

Soap film equilibrium in a tetrahedral frame

$C = 22.794$  vertices



- Pentagonal dodecahedron (12 pentagons)  
with 20 corners (close-packing of spheres)
- Tetrakaidecahedron (Regularly truncated octahedron)  
with 24 corners (bcc packing)  
(8 hexagons and 6 squares)



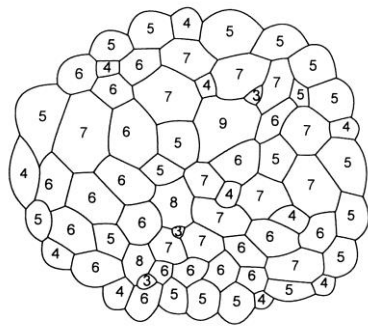
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# Topological Relationship

Euler’s Law describes the relationship btw. features of different dimensionality

$$n_0 - n_1 + n_2 - n_3 = 1 \text{ ( C-E+P-B=1)}$$
 eg.

- 2-dim:  
av. number of sides of polygons = 6



Case of NGG  
AGG

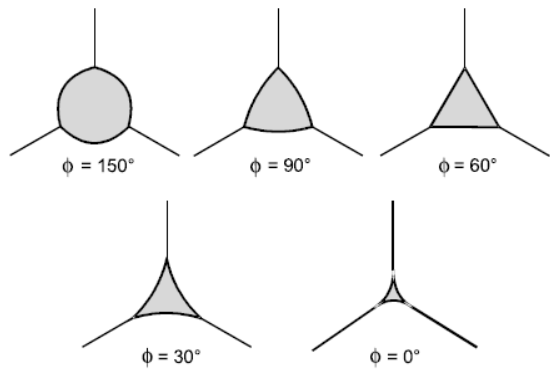
- 3-dim:  
av. number of sides of polygons < 6

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## 2-Phase Microstructure

Shape of a second phase =  $f(\phi)$

In 2-dim



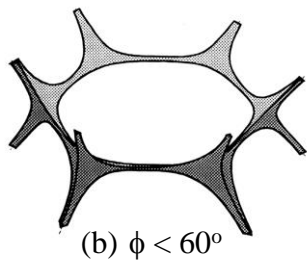
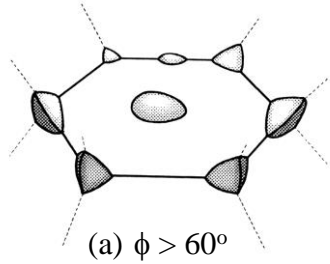
ex. Dihedral angle and physical properties  
Qn: Distribution?

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# 2-Phase Microstructure

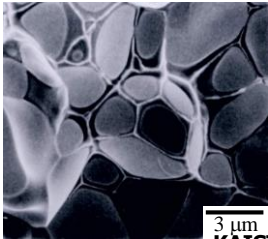
Shape of a second phase =  $f(\phi)$

In 3-dim

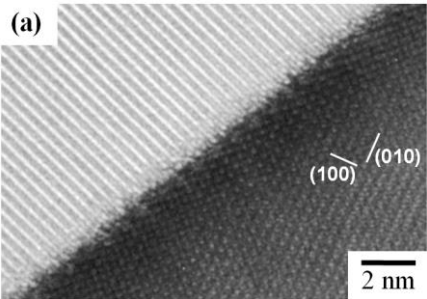


Qn: Measurement of dihedral angle?

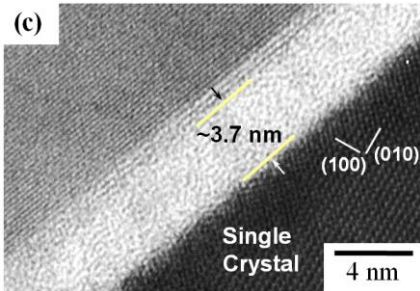
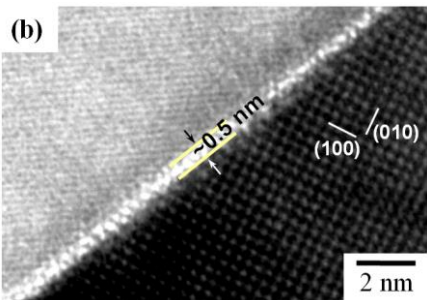
Qn: Thickness of liquid film?



3  $\mu\text{m}$   
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Kinetic thickening of liquid film



**Figure 3-12.** HREM images of the boundaries between a single crystal and a fine matrix grain. (100) single crystal/polycrystal bi-layer samples with 0.4-mol%-TiO<sub>2</sub> addition annealed at 1350°C for (a) 5, (b) 20, and (c) 50 h in air after H<sub>2</sub>-treatment at 1250°C for 10 h.<sup>22</sup>

Choi, et al., *Acta Mater.*, **52**, 3721 (2004)

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## 2-Phase Microstructure

### *Equilibrium Microstructure*

Qn: Conditions that govern the equilibrium microstructure of mono-size grains?

Qn: Effective pressure of a powder compact?

eg) Elimination of pores during sintering

Redistribution of liquid between two samples in contact with different amounts of liquid

Minimum Interfacial Energy Configuration =  $f(\phi, \text{amt of matrix})$

For an infinitesimal change under equilibrium,

work done =  $P_e \cdot dV_m$

= total interfacial energy change ( $\Delta(\gamma A)$ )

$$P_e = - \frac{(1 - f_m)^2}{V_g} \left[ \frac{\partial E}{\partial f_m} \right]_{V_g}$$

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### Calculated variation of total interfacial energy

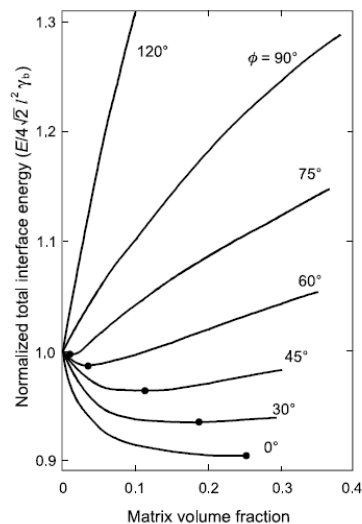


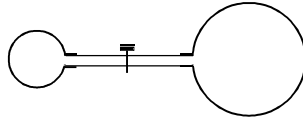
Figure 3.13. Calculated variation of total interfacial energy,  $E$ , with the matrix volume fraction (at constant grain volume) for various dihedral angles.<sup>24</sup> The minimum  $E$  values are shown by filled circles.

H. H. Park and D. N. Yoon, *Metall. Trans. A*, 16A, 923 (1985)

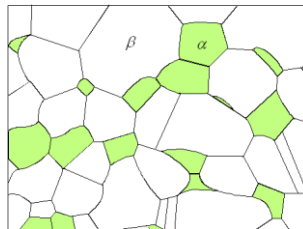
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## Exercises:

- Elastic balloons



- Equilibrium shape of an entrapped pore
- Measurement of  $\gamma_{\alpha/\alpha} / \gamma_{\beta/\beta}$



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